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Report on the AFOSR-IFP-Stanford conference on computation of turbulent boundary layers

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A conference on turbulent boundary-layer prediction was held at Stanford, California, from 18 to 23 August 1968. The meeting was sponsored by the Mechanics Division of the U.S. Air Force Office of Scientific Research and by the industrial sponsors of the Internal Flow Program of the Mechanical Engineering Department at Stanford University. Attendance was by invitation, and there were 75 participants. The following is a brief account of the organization and content of the meeting; the full proceedings are available in two volumes.†

1. Introduction; the purpose and organization of the meeting

The development of high-speed computers has led to a proliferation of computational methods for predicting the development of turbulent boundary layers, starting from given conditions at some initial section, and for a given external pressure distribution. The methods are all based to some extent on the Navier–Stokes equations in the boundary-layer approximation. Time-averaging of these equations (and of any moments of them) leads to an excess of unknown quantities over equations to determine them, and the prediction methods differ only in the means by which further relations between the unknown quantities are obtained in order to make the system of equations determinate. These correlations cannot be deduced from the Navier–Stokes equations, and they are at best an approximation to the truth; they are necessarily the result of a combination of guesswork (suitably guided by 'physical intuition') and of careful inference from experiments over limited ranges of the parameters involved. Ideally, modern informa-

† These have been reproduced by direct photo offset process and may be obtained from the Mechanical Engineering Department, Stanford University, Stanford, California 94305, for $6.00 each, or $11.00 for the set of two, plus postage (U.S. 50c, Europe $1.00); hard covers $1.00 extra per volume.

tion on the structure of turbulent boundary layers should be directly incorporated, but at present few of the methods use even a small part of this information as a guide to the form of the assumed additional relations.

The number and variety of prediction procedures creates acute problems of choice for potential users, and it was felt that it would be useful to provide a comparative evaluation of the various procedures, particularly in terms of their accuracy, computational speed, and adaptability to widely varying conditions. The Stanford meeting was organized with this as its prime objective. The organizing committees were formed in early 1967; these were an Executive Committee consisting of D. E. Coles, M. V. Morkovin (Chairman), and G. Sovran; an International Advisory Board consisting of F. H. Clauser, H. W. Emmons, H. P. Liepmann, J. C. Rotta, and I. Tani; and a Stanford host committee consisting of E. A. Hirst, S. J. Kline (Chairman) and W. C. Reynolds.

It was first necessary to select and standardize a suitable body of experimental data for the testing of prediction procedures. This task was undertaken by Coles and Hirst; it was found at an early stage (i) that sufficient reliable data for the purpose existed only for two-dimensional, incompressible, turbulent boundary layers, and (ii) that, even for these, complete restandardization was necessary, since discrepancies as large as 30 % in some parameters could arise solely through variations in the methods by which various workers had handled the data. Experimental results for 33 boundary layers were standardized, using some improvements on Coles' well-known correlation procedures. In particular, the dependence of the Reynolds number R_θ (based on momentum thickness), the shape factor H, and the friction coefficient C_f on the streamwise co-ordinate was determined for each boundary layer. This body of standard data is available as volume II of the Proceedings.

In a separate paper pointedly entitled 'The Young Person's Guide to the Data', Coles has noted a number of reasons why the data are not entirely sufficient in scope and quality even for the two-dimensional, incompressible case. Among the many factors which contribute to a high noise level in the available data are effects of: probe size, tripping devices, and wall curvature; uncertainty in probe corrections for fluctuations; lack of redundant checks; lack of adequate two-dimensionality; and the dominance of a few points very near the wall in determining several integral parameters. If certifiable progress is to be achieved towards higher accuracy of prediction for this basic class of flows and towards extensions to other important cases, more extensive and better co-ordinated experimental efforts are needed and these should include careful cross-checking and standardization of results.

Invitations were sent to all workers known to have a potentially viable prediction method, and all except three were ultimately able to attend. In all, 30 methods were represented at the meeting, of which three were older methods programmed by Stanford students for comparative purposes. Tabulated data defining 33 standard flows were sent to the predictors, who were required to predict the development of the quantities R_θ, H and C_f for the flows, each according to his own method; 16 of the flows were mandatory, the rest optional. The mandatory flows include decelerating, accelerating, zero-pressure gradient, equilibrium, non-equilibrium and reattaching boundary-layers. The predicted results were replotted, in a manner that facilitated comparison; there were about 3000 curves in all, and these were distributed at the start of the meeting, together with preprints of the papers to be presented. Check runs on all but two methods (which arrived late) were also made by Stanford students to ensure completeness and repeatability.

In addition to the predictors, all workers who were known to have taken relevant data, and many who have done important work on shear flow structure problems, were invited, and most were able to attend.

The first 3 days of the meeting were devoted to the presentation and discussion of the various prediction methods. After a day off, the final 2 days were devoted to evaluation of the methods, to general discussion and to papers dealing with the underlying physics of the turbulence. All discussions were recorded, and were reduced for inclusion in the proceedings by editing committees under the general supervision of Sovran.

2. The prediction methods

These were classified according to a scheme prepared by Reynolds. There were 21 'integral' methods, which in all cases required the numerical integration of coupled *ordinary* differential equations and algebraic equations; all such methods used the 'global' momentum equation obtained by integrating the mean momentum equation across the boundary layer, together with at least one other ordinary differential equation (e.g. an equation describing the rate of entrainment of non-turbulent fluid, or an equation representing an over-all turbulent energy balance). The algebraic equations arose, for example, from assumptions concerning the mean velocity profile in the inner and outer regions (e.g. the law of the wall and the law of the wake) or from assumptions relating turbulent quantities to the mean velocity profile (e.g. a formula for eddy viscosity).

The other 9 methods were 'differential' methods which required the numerical integration of coupled *partial* differential equations and algebraic equations. Such methods, at least in principle, are much more powerful than the integral methods (they can, for example, predict the development of the mean-velocity profile, as well as of integral parameters such as R_θ, H and C_f), but they are also much more extravagant in terms of computer time, and they require more input. Six of these methods used an eddy viscosity and/or mixing length assumption; the other three used 'structure assumptions' relating Reynolds stress to turbulence intensity.

The predicted results were studied by a special evaluation committee consisting of D. J. Cockrell, H. W. Emmons (Chairman), P. G. Hill, J. L. Lumley, and M. V. Morkovin. The committee produced a rough 'order of merit' dividing all the methods into three groups based on the accuracy with which the 16 mandatory flows had been predicted. The committee also expressed a number of views, including the following: (i) the best dozen or so of these methods perform very well—essentially as well as can be expected in view of the uncertainties in the available data; (ii) methods ranking in the lowest third and not capable of significant improvement should be abandoned, and further proliferation of two-dimensional incompressible methods without clearly superior features is undesirable; (iii) the better integral methods predict R_θ, C_f and H for the mandatory flows essentially as well as the better differential methods; (iv) no preferred physical or mathematical framework can be inferred from the results; performance in predicting the mandatory flows seems to depend more on the skill of interpolating and on the use of sufficient data than on mathematical or physical insight; and (v) there is an important continuing need for more accurate and extensive experimental data. In some summary comments by the editors of volume I, conclusions (i) and (ii) of the evaluation committee are further emphasized, and a list is provided of desirable extensions to other classes of problems and to known physical effects not now incorporated in any method; it is noted that both types of extensions will require more and better data. The editors also remark that most of the more successful methods do incorporate the law of the wall and the law of the wake, or equivalent information, while many of the less successful methods do not.

3. The underlying physics of the turbulence; conclusions

Throughout the meeting it was repeatedly stressed that the ultimate success of any prediction method depends on how faithfully it reflects the 'underlying physics of the turbulence', i.e. on the extent to which it retains the detailed information conveyed by the Navier–Stokes equations, while yet reducing them by judicious approximation to tractable form. Two final sessions were devoted to a consideration of topics that have a bearing on this underlying physics. In the first session, ideas concerning interpretation of structure measurements were discussed by L. S. G. Kovasznay, J. Sternberg and S. J. Kline. For the proceedings these presentations, and the resulting discussions, were reduced to summary form with special attention to points on which there is clear agreement or strong disagreement. In the second session, M. J. Lighthill interpreted the eruption of disturbances from the viscous sublayer in terms of propagation of wave-packets in a medium of non-uniform mean velocity; H. K. Moffatt explored the wave-eddy duality in the outer part of the turbulent boundary-layer; and finally, I. Tani reviewed some experimental results on the response of a turbulent boundary layer to sudden perturbations (e.g. a sudden change of wall roughness, or a sudden change of pressure gradient). These papers are included in volume I of the Proceedings, together with the prediction papers, the recorded discussions, and the reports of the various committees.

The Proceedings, which reflect to a large extent the co-operative effort of all who attended, will be of value to users in resolving the problem of choice of prediction procedure and to researchers as a basis for further investigations. The conference has drawn attention to the need for increased experimental effort, and for more attention to co-ordination, checking and standardization of experimental results.